Endogenous Regime Switching

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Broad questions

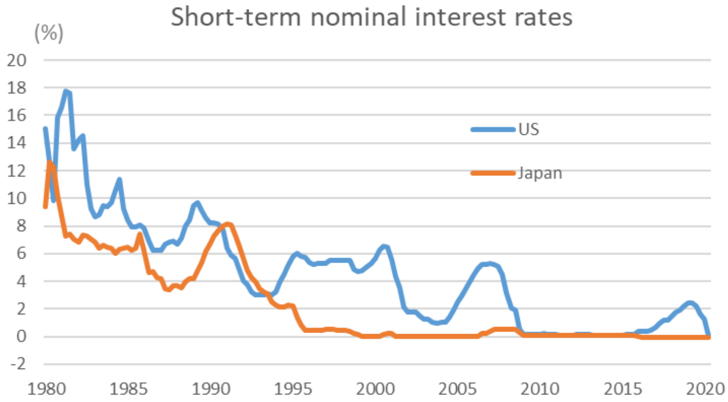
How can we effectively model endogenously switching regimes to account for . . .

1. Occasionally binding constraints ("Zero Lower Bound")?
2. Asymmetries (recessions versus expansions)?
3. Tipping points (financial distress, debt sustainability)?

- Voluminous literatures on various types of *exogenous* regime switching
- But work on *endogenous* regime switching limited
- Why is this interesting?

Motivation: the zero lower bound

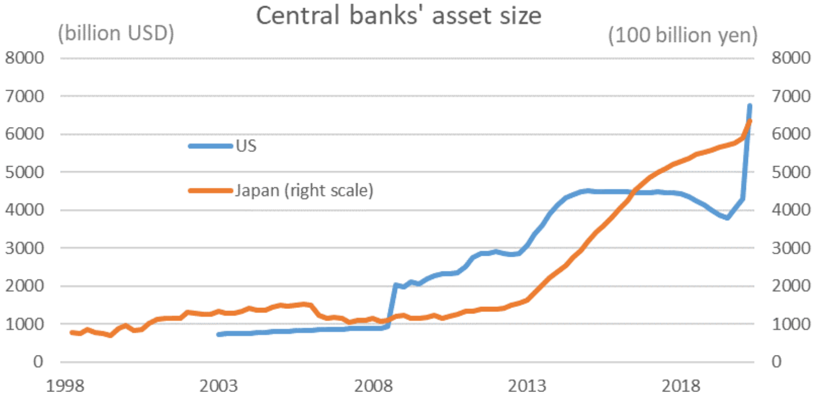
Short-term interest rate has been the primary monetary policy instrument in Japan and the US



- Policy rates have spent long periods at or near the zero lower bound
- How can we model this using something akin to a structural VAR?

Unconventional monetary policy (UMP)

- Forward guidance (FG) – commitment about future interest rates
- Quantitative easing (QE) – purchases of long-term government bonds



A nonlinear structural VAR

- These questions cannot be answered by linear SVARs

$$A_{0,t} z_t = c_t + \sum_{i=1}^{k} A_{i,t} z_{t-i} + \varepsilon_t$$

- even with exogenous regime switching
- They can be answered by additively time-separable nonlinear SVARs:

$$f_0(z_t) = c + \sum_{i=1}^{k} f_i(z_{t-i}) + u_t$$

- When $f_i(\cdot)$ are piecewise linear, they represent endogenously-switching regimes

Overview

1. Endogenous versus exogenous regime switching
2. General model setup
3. Identification
4. Estimation
5. Applications

Background: methodology

- Question: causal effect of policy on targets
  - x_t: policy instrument, e.g., short-term nominal interest rate
  - y_t: policy targets, e.g., inflation, output

- Methodology: Structural Vector Autoregression (SVAR)

$$\sum_{j=0}^{p} A_j z_{t-j} = \varepsilon_t, \qquad z_t := \begin{bmatrix} x_t \\ y_t \end{bmatrix}$$

  - Causal effects of policy given by impulse response function (IRF)

- Problem: linear SVAR cannot capture ZLB
  - Not structural change or Markov Switching
  - Need endogenous regime switching

Why exogenous regimes don't work

- Consider regime-switching VAR

$$z_t = \sum_{j=1}^{p} C_j^{(r_t)} z_{t-j} + C_0^{(r_t)} \varepsilon_t$$

- Try to impose the ZLB by having two exogenous regimes $r_t \in \{1, 2\}$
  - For simplicity, assume z_t is scalar
  - We can enforce $z_t = 0$ by choosing $C_j^{(1)} = 0$ for all $j \in \{0, \ldots, p\}$
  - Cannot guarantee $z_t > 0$ in regime 2, unless r_t is *perfectly correlated* with ε_t
  - Otherwise, there will be realizations of ε_t that violate the constraint
- This rules out:
  - Markov-Switching (Hamilton, 1989) strictly exogenous: $r_t \perp \varepsilon_s$ for all t, s
  - predetermined regimes (Hubrich & Waggoner, 2022), $\text{corr}(r_t, \varepsilon_s) \neq 0, s < t$

Solution: fully endogenous regimes

- Go back to the linear SVAR

$$\sum_{j=0}^{p} A_j z_{t-j} = \varepsilon_t \tag{1}$$

- and imagine **the variable z_t itself** is driving the regime
- Specifically, you only observe $z_t^+ = \max\{z_t, 0\}$
- Now, the regime indicator is ***perfectly correlated*** with ε_t

$$r_t = \begin{cases} 1 & \text{if } z_t \leq 0 \text{ (ZLB regime)} \\ 2 & \text{if } z_t > 0 \text{ (positive regime)} \end{cases}$$

- The **same shock** drives $z_t^+ > 0$ and determines probability of $z_t^+ = 0$

The CKSVAR model

$$\text{VAR}: \qquad \sum_{j=0}^{p} A_j z_{t-j} = \varepsilon_t$$

- Mavroeidis (2021, Ecma): the censored and kinked structural VAR:

$$A_j z_{t-j} = \begin{bmatrix} A_{j,x} & A_{j,y} \end{bmatrix} \begin{bmatrix} x_{t-j} \\ y_{t-j} \end{bmatrix} \to \begin{bmatrix} A_{j,x}^{+} & A_{j,x}^{-} & A_{j,y} \end{bmatrix} \begin{bmatrix} x_{t-j}^{+} \\ x_{t-j}^{-} \\ y_{t-j} \end{bmatrix}$$

$$\text{CKSVAR:} \qquad \sum_{j=0}^{p} A_{j,x}^{+} x_{t-j}^{+} + \sum_{j=0}^{p} A_{j,x}^{-} x_{t-j}^{-} + \sum_{j=0}^{p} A_{j,y} y_{t-j} = \varepsilon_t$$

(cf. Aruoba, Mlikota, Schorfheide & Villalvazo, 2022, JoE)

- $x_t^{+} := \max\{x_t, b\}$ and $x_t^{-} := \min\{x_t, b\}$ (assume x_t is scalar)
- x_t enters nonlinearly, with coefficients depending on its sign
- Two endogenously switching regimes

Challenges

1. Theoretical

    - Can the CKSVAR be microfounded? Does it have economic interpretation?
        - Yes: Ikeda Li Mavroeidis & Zanetti (2024, AEJM) ▶

    - Econometrics
        - Identification (Mavroeidis, 2021, Ecma; Guarnieri, 2024)
        - Trends and cointegration (Duffy Mavroeidis & Wycherley, 2023a,b; Duffy and Mavroeidis, 2024)

2. Practical

    - Estimation is computationally intensive and inaccurate

    - New analytical algorithm for computing likelihood (Bonomolo, Guarnieri, Kabel, and Mavroeidis, 2024)
        - Implemented by Julia package EndoRSE (stands for Endogenous Regime-Switching model Estimation)

General piecewise affine state-space model

- $z_t \in \Re^n$ (latent) state variables

$$f_{0,t}(z_t) = c_t + \sum_{i=1}^{p} f_{i,t}(z_{t-i}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, I_n)$$

$$f_{i,t}(z) = \sum_{r=1}^{R} 1_{t-i}^{(r)}(z) \left(\psi_{i,t}^{(r)} + \Psi_{i,t}^{(r)} z \right), \quad i = 0, \ldots, p$$

where $1_s^{(r)}(z) := 1\left\{ z \in \mathcal{Z}_s^{(r)} \right\}$, $\left\{ \mathcal{Z}_s^{(r)} \right\}_{r=1}^{R}$ is a (convex) partition of $\Re^n$

- R: number of regimes, ε_t structural shocks
- $y_t \in \Re^k$ observed variables

$$y_t = \sum_{r=1}^{R} 1^{(r)}(z) \left(\mathrm{g}_t^{(r)} + \mathrm{G}_t^{(r)} z_t \right)$$

Why piecewise affine functional form

$$f_{0,t}(z_t) = c_t + \sum_{i=1}^{p} f_{i,t}(z_{t-i}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, I_n)$$

$$f_{i,t}(z) = \sum_{r=1}^{R} 1_{t-i}^{(r)}(z) \left(\psi_{i,t}^{(r)} + \Psi_{i,t}^{(r)} z \right), \quad i = 0, \ldots, p$$

- We can impose continuity and verify coherency (more later)
- Fully characterize stochastic trends and cointegration
  - Duffy Mavroeidis and Wycherley (2023)
- Derive short-run and long-run identifying restrictions
  - Duffy and Mavroeidis (2024)
- Good approximation to more general nonlinear DSGEs
  - Aruoba et al (2021, RED)

Notable examples

- Single-regime ($R = 1$) (linear) models

$$\Psi_{0,t} z_t = c_t + \sum_{i=1}^{p} \Psi_{i,t} z_{t-i} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, I_n)$$
$$y_t = g_t + G_t z_t$$

- e.g., linear SVAR: $n = k$, $g_t = 0$, $G_t = I_k$
- Laumbach and Williams (2003, RES) model of natural rate: $n > k$, $\Psi_{i,t} = \Psi_i$, $g_t = 0$
- ...

Notable examples

- Models with two regimes $r \in \{1, 2\}$ determined by inequality:

$$r_t = \begin{cases} 1, & \text{if } a_1^\top z_t \leq b_t \\ 2, & \text{if } a_1^\top z_t > b_t \end{cases}$$

- e.g., debt sustainability: $z_t =$[GDP growth, long-term interest rate,...], $a_1^\top = (1, -1, 0, \ldots)$, $b_t = 0$

- ZLB: $z_t =$[policy rate, ...], $a_1^\top = (1, 0, \ldots)$, $b_t = 25$bp in the US, or linked to IOR in Japan (Hayashi and Koeda, 2019, QE)

Notable examples

- Models with three regimes $r \in \{1,2,3\}$ determined by inequalities

$$r_t = \begin{cases} 1, & \text{if } a_1^\top z_t \le \underline{b}_t \\ 2, & \text{if } \underline{b}_t < a_1^\top z_t \le \overline{b}_t \\ 3, & \text{if } a_1^\top z_t > \overline{b}_t \end{cases}$$

- e.g., inflation target band $[0, 2\%]$ (ECB, Fed): $z_t =$[inflation, ...], $a_1^\top = (1, 0, \dots)$, $\underline{b}_t = 0$, $\overline{b}_t = 2\%$

Notable examples

- Models with four regimes $r \in \{1,2,3,4\}$ determined by two inequalities

$$r_t = \begin{cases} 1, & \text{if } a_1^\mathsf{T} z_t \leq b_{1t}, \text{ and } a_2^\mathsf{T} z_t \leq b_{2t} \\ 2, & \text{if } a_1^\mathsf{T} z_t \leq b_{1t}, \text{ and } a_2^\mathsf{T} z_t > b_{2t} \\ 3, & \text{if } a_1^\mathsf{T} z_t > b_{1t}, \text{ and } a_2^\mathsf{T} z_t \leq b_{2t} \\ 4, & \text{if } a_1^\mathsf{T} z_t > b_{1t}, \text{ and } a_2^\mathsf{T} z_t > b_{2t} \end{cases}$$

- e.g., ZLB and "high inflation" regime $z_t = [i_t, \pi_t, \ldots]^\mathsf{T}$, $a_1^\mathsf{T} = (1, 0, \ldots)$, $a_1^\mathsf{T} = (0, 1, 0, \ldots)$, $b_{1t} =$ELB, $b_{2t} = \bar{\pi}$

- asymmetries, tipping points, ...

Identification via regime switching

- Regime-switching generates more variation in the data
  - Dynamics and volatilities change across regimes
- This can be used to identify causal effects
- Special case: identification at the zero lower bound (Mavroeidis, 2021)
  - Main insight: change in regime is only due to policy constraint
  - Imposing single regime corresponds to "ZLB irrelevance" hypothesis that UMP is fully effective in escaping the liquidity trap
  - Variation across regimes ("ZLB relevance") is informative about the relative effectiveness of conventional and unconventional policies

Identification in linear SVAR

- Linear SVAR identified up to orthogonal rotation $\Upsilon \in \mathbb{R}^{k\times k}$

$$\Psi_0 z_t = c + \sum_{i=1}^{p} \Psi_i z_{t-i} lags + \varepsilon_t, \qquad \varepsilon_t \sim_{\text{i.i.d.}} [0, I_k]$$

(suffices to focus on variance)

- Reduced form:

$$z_t = lags + \underbrace{\Psi_0^{-1}\varepsilon_t}_{=:u_t}, \qquad \Omega := \text{var}\, u_t = \Psi_0^{-1}\Psi_0^{-1\mathsf{T}}$$

- Reduced-form parameters $\Omega : k\,(k+1)\,/2$
- Structural parameters $\Psi_0 : k^2$
- Need $k\,(k-1)\,/2$ additional restrictions to pin down Ψ_0 uniquely
- Can identify some shocks/IRFs with fewer restrictions
  - e.g., identify ε_1 by imposing $\frac{\partial z_{\mathbf{2}t}}{\partial \varepsilon_{\mathbf{1}t}} = 0 \Leftrightarrow \Psi_{0,21} = 0$ ($k-1$ restrictions)

Identification in SVAR with endogenous regime switching

- Introduce endogenous regimes $r \in \{1, \ldots, R\}$

$$\Psi_0^{(r_t)} z_t = lags + \varepsilon_t, \qquad \varepsilon_t \sim_{\text{i.i.d.}} [0, I_k]$$

- Focus on regimes determined by signs of z_t (wlog upon recentering)
  - 2^m regimes induced by $z_{i,t} \geq 0$, $i = 1, \ldots, m \leq k$
- For coherency (existence of equliibria) we need piecewise linear function $\Psi_0^{(r)} z$ to be invertible
- This requires continuity and some determinant condition
  - Gourieroux et al (1980, Ecma)
- Continuity requires **only** coefficients on $z_{i,t}$ change when $z_{i,t}$ changes sign
- CKSVAR is special case with $m = 1$

CKSVAR

- Partition $z_t = \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} \begin{matrix} 1 \\ k-1 \end{matrix}$ such that

$$\overset{k\times k}{\Psi_0^{(r_t)}} z_t = \overset{k\times 1}{\Psi_{0,1}^{-}} z_{1,t}^{-} + \overset{k\times 1}{\Psi_{0,1}^{+}} z_{1,t}^{+} + \overset{k\times(k-1)}{\Psi_{0,2}} z_{2,t}$$

- Only first column of $\Psi_0^{(r)}$ changes across regimes

$$\Psi_0^{(1)} = \left[\Psi_{0,1}^{-}, \Psi_{0,2}\right], \quad \Psi_0^{(2)} = \left[\Psi_{0,1}^{+}, \Psi_{0,2}\right]$$

- This is crucial for continuity at the kink $z_{1,t} = 0$
- Total # of structural parameters $\Psi_0^{(1,2)}$: $k\,(k+1)$
- Now let's look at the reduced form...

CKSVAR: reduced form

- Structural model

$$\Psi_0^{(r_t)} z_t = \ldots + \varepsilon_t, \quad \Psi_0^{(1)} = \left[\Psi_{0,1}^{-}, \Psi_{0,2}\right], \quad \Psi_0^{(2)} = \left[\Psi_{0,1}^{+}, \Psi_{0,2}\right]$$

- Reduced form

$$z_t = \begin{cases} \ldots + \left(\Psi_0^{(1)}\right)^{-1} \varepsilon_t = \left(\Psi_0^{(1)}\right)^{-1} \left(\Psi_0^{(2)}\right) u_t =: \tilde{u}_t, & \text{if } z_{1,t} \leq 0 \\ \ldots + \left(\Psi_0^{(2)}\right)^{-1} \varepsilon_t =: u_t, & \text{if } z_{1,t} > 0 \end{cases}$$

- Let $\Omega := \text{var}\, u_t = \left(\Psi_0^{(2)}\right)^{-1} \left(\Psi_0^{(2)}\right)^{-1\mathsf{T}}$ (variance in “positive” regime)
- It can be shown that $\left(\Psi_0^{(1)}\right)^{-1} \left(\Psi_0^{(2)}\right) =: B = \begin{bmatrix} \kappa & 0 \\ -\tilde{\beta} & I_{k-1} \end{bmatrix}$
  - κ scales variance of $z_{1,t}$ across regimes $\tilde{u}_{1,t} = \kappa u_{1,t}$
  - $\tilde{\beta}$ is “kink” in $\tilde{u}_{2,t} = u_{2,t} - \tilde{\beta} u_{1,t}$
- Thus, $\tilde{\Omega} := \text{var}\, \tilde{u}_t = B\Omega B^{\mathsf{T}}$– (only) k additional parameters

CKSVAR: identification

- Total # of structural parameters $\left[\Psi_{0,1}^{-}, \Psi_{0,1}^{+}, \Psi_{0,2}\right]$: $k^2 + k$
- Total # of reduced-form parameters $\Omega, \tilde{\Omega}$: $k(k+1)/2 + k$
- Underidentified by $k(k-1)/2$, exactly as in linear case:
  - Adding more regimes does not make identification problem worse!
  - Suffices to impose restrictions in one regime only to identify **both** regimes
- In fact, regime-switching is informative about $\left[\Psi_{0,1}^{-}, \Psi_{0,1}^{+}\right]$ even without additional restrictions

$$\tilde{\beta} = \kappa \Psi_{0,22}^{-1} \Psi_{0,21}^{-} - \Psi_{0,22}^{-1} \Psi_{0,21}^{+}$$

- Thus, $\Psi_0^{(1)} = \Psi_0^{(2)} \Rightarrow \kappa = 1, \tilde{\beta} = 0$ testable!
  - "ZLB irrelevance" hypothesis (Ikeda et al, 2024, AEJM)
- With some additional structure, we can get meaningful set identification of IRFs to $\varepsilon_{1,t}$ (Mavroeidis, 2021)

A more general result

- We suppose there exist $(c^{\varepsilon}, \{f_i^{\varepsilon}\}_{i=0}^{p})$ such that

$$f_0^{\varepsilon}(z_t) = c^{\varepsilon} + \sum_{i=1}^{p} f_i^{\varepsilon}(z_{t-i}) + \varepsilon_t \qquad \varepsilon_t \sim_{\text{i.i.d.}} [0, I_k]$$

  where $\{\varepsilon_t\}$ are the *structural* shocks
- Let $(c, \{f_i\}_{i=0}^{p})$ be another parametrisation of the model, such that

$$f_0(z_t) = c + \sum_{i=1}^{p} f_i(z_{t-i}) + \eta_t \qquad \eta_t \sim_{\text{i.i.d.}} [0, I_k]$$

- When are these observationally equivalent? If *and only if*

$$c = \Upsilon c^{\varepsilon} \qquad f_i(z) = \Upsilon f_i^{\varepsilon}(z),\ \forall z \in \mathbb{R}^k,\ i \in \{0, \dots, p\}$$

  for some orthogonal $\Upsilon \in \mathbb{R}^{k \times k}$ (via Matzkin, 2008)
- Duffy and Mavroeidis (2024, Appendix A)

Specializing to endogenous regime switching SVARs

- The previous result assumes $f_0()$ is a homeomorphism (continuous and invertible)
- It covers piecewise affine continuous SVARs, e.g.,

$$f_0(z) = \sum_{r=1}^{R} 1^{(r)}(z)\, \Psi_0^{(r)} z,$$

where regimes $r = 1, \dots, 2^m$ are determined by signs of z_i, $i = 1, \dots, m$

- Thus, $\Psi_0^{(1)}, \dots, \Psi_0^{(2^m)}$ are identified up to (**the same**) orthogonal rotation (across regimes)
- It suffices to impose exactly $k(k-1)/2$ restrictions to identify **all** the causal effects

Practical challenges

- Estimation of models with endogenous regime switching is likelihood based
  - We need to know the distribution of the data to characterize the (endogenous) regime-switching probability
- A key challenge is the presence of latent variables on the RHS
  - These need to be 'integrated-out' of the likelihood
  - Currently, all available methods rely on Monte Carlo simulations (particle filtering)
    - Mavroeidis (2021, Ecma), Johansen and Mertens (2021, JMCB), Aruoba et al (2022, JoE), Carriero et al (2024)
  - These are computationally expensive and suffer from problems of particle degeneracy
- We found analytical expression of likelihood involving Gaussian cdfs
  - Massively reduces computational time and increases accuracy
- Developped Julia program called EndoRSE, to facilitate application of these methods

Illustration: univariate censored-kinked AR

- $k = \dim z_t = 1$, $z_t^+ := \max\{z_t, 0\}$, $z_t^- := \min\{z_t, 0\}$

$$\Psi_0^{(1)} z_t^- + \Psi_0^{(2)} z_t^+ = c + \Psi_1^{(1)} z_{t-1}^- + \Psi_1^{(2)} z_{t-1}^+ + \varepsilon_t$$
$$y_t = z_t^+$$

- e.g., z_t indicator of MP stance, $y_t = i_t$ is policy rate, $z_t^- := \min\{z_t, 0\}$ is "shadow rate" (measures of UMP)
- $\Psi_i^{(1)} \neq \Psi_i^{(2)}$ means behaviour (volatility, dynamics) changes across regimes (this is testable)

Likelihood

$$\Psi_0^{(1)} z_t^- + \Psi_0^{(2)} z_t^+ = c + \Psi_1^{(1)} z_{t-1}^- + \Psi_1^{(2)} z_{t-1}^+ + \varepsilon_t$$
$$y_t = z_t^+$$

- Likelihood given sample $Y_T = (y_1, \ldots, y_T)^\mathsf{T}$:

$$f(y_1, \ldots, y_T; \theta) = \prod_{y_t > 0} \int f\left(y_t | Z_{t-1}^+, Z_{t-1}^-; \theta\right) f\left(Z_{t-1}^- | Z_{t-1}^+; \theta\right) dZ_{t-1}^-$$
$$\times \prod_{y_t = 0} \int F\left(0 | Z_{t-1}^+, Z_{t-1}^-; \theta\right) f\left(Z_{t-1}^- | Z_{t-1}^+; \theta\right) dZ_{t-1}^-$$

- Difficulty lies in evaluating the integrals
  - Number of latent lags $\dim Z_{t-1}^-$ depends on number of (consecutive) observations at boundary
  - As $\dim Z_{t-1}^-$ increases, simulation methods become very inaccurate (particle degeneracy)
  - Quickly gets computationally infeasible as $\dim z$ increases (many variables)
  - Our solution: we discovered an analytical expression for the integrals!

Monetary policy transmission in the Euro Area

- Check the empirical relevance of non linearities that can affect monetary policy
- Questions:
  - Is the effective lower bound a cost for the EA economy?
  - Are the policy trade-offs different when inflation is high? (Non-linear Phillips curve)
  - Are risks of de-anchoring dependent on the level of inflation?

A model with 4 regimes

- Variables: (EA) short-term interest rate, HICP inflation, and output gap.
- Four regimes determined by
  - whether ELB constraint binds ($z_{1t} \leq 0$ regimes 1 and 3) or not ($z_{1t} > 0$ regimes 2 and 4)
  - whether inflation is "low" ($z_{2t} \leq 0$ regimes 1 and 2) or high ($z_{2t} > 0$ regimes 3 and 4)
- Observed interest rate: $z_{1t}^{+} + b_1$ (b_1 is ELB)
- Inflation is $z_{2t} + b_2$ (fully observed, b_2 estimated from the data)

$$\Psi_i^{(r)} = \begin{pmatrix} \Psi_{i,11}^{(r)} & \Psi_{i,12}^{(r)} & \Psi_{i,13} \\ \Psi_{i,21}^{(r)} & \Psi_{0,22}^{(r)} & \Psi_{i,23} \\ \Psi_{i,31}^{(r)} & \Psi_{i,32}^{(r)} & \Psi_{i,33} \end{pmatrix}, \quad r = \begin{cases} 1 & \text{if } \{\text{ELB \& Low } \pi\}, \\ 2 & \text{if } \{\text{Non ELB \& Low } \pi\}, \\ 3 & \text{if } \{\text{ELB \& High } \pi\}, \\ 4 & \text{if } \{\text{Non ELB \& High } \pi\}. \end{cases}$$

- Only first 2 columns of $\Psi_i^{(r)}$ are regime dependent (necessary for continuity)

Coherency and identifying restrictions

Coherency Conditions:

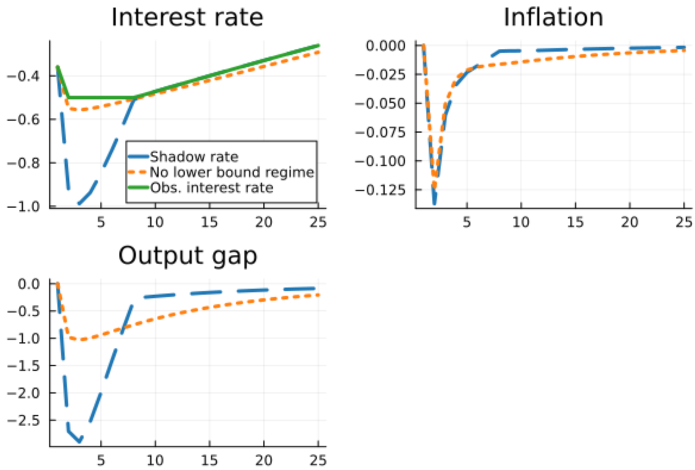
- $\Psi^{(1)}_{0,\cdot 1} = \Psi^{(3)}_{0,\cdot 1} = \Psi^{-}_{0,\cdot 1}$ (ELB regimes)
- $\Psi^{(2)}_{0,\cdot 1} = \Psi^{(4)}_{0,\cdot 1} = \Psi^{+}_{0,\cdot 1}$ (non-ELB regimes)
- $\Psi^{(1)}_{0,\cdot 2} = \Psi^{(2)}_{0,\cdot 2} = \Psi^{-}_{0,\cdot 2}$ (low inflation regimes)
- $\Psi^{(3)}_{0,\cdot 2} = \Psi^{(4)}_{0,\cdot 2} = \Psi^{+}_{0,\cdot 2}$ (high inflation regimes)
- $\det \Psi^{(r)}_{0}$ have the same sign for all $r \in \{1, ..., 4\}$

Identifying Restrictions:

- $\Psi^{-}_{0,11} = \Psi^{+}_{0,11}$ (normalization)
- $\Psi^{-}_{0,j1} = 0$ for $j = \{2, 3\}$ (UMP ineffective on impact)
- $\Psi^{-}_{i,32} = \Psi^{+}_{i,32}$ for $i = \{0, ..., p\}$ (output response to inflation constant across regimes)

Implications of the ELB

Simulation: a negative demand shock brings the interest rate at the lower bound



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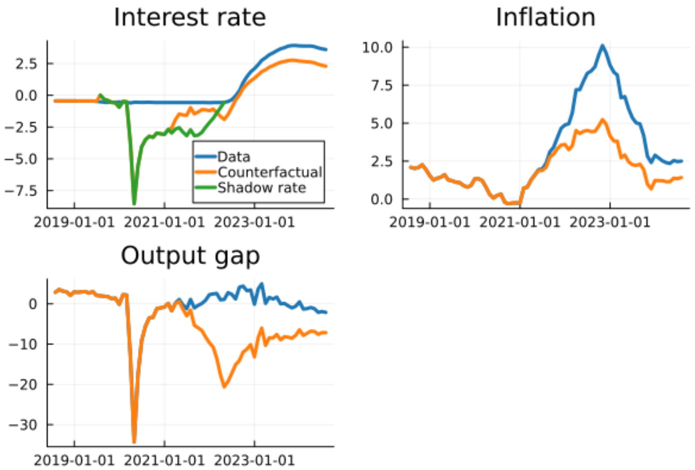
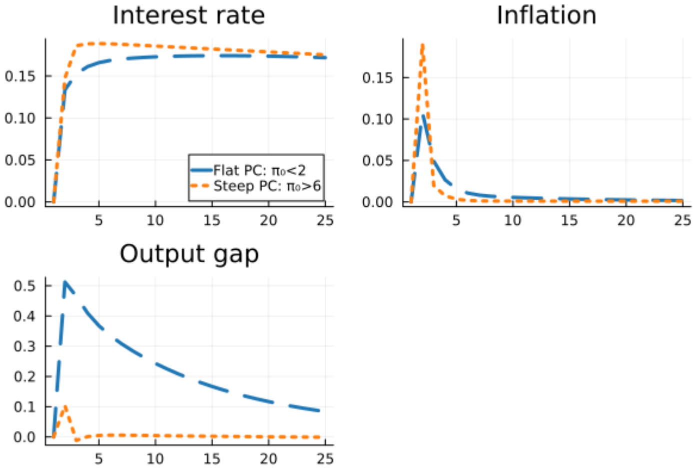
[1] Inflation responses are month on month

Scenario: less accommodating monetary policy in the aftermath of the pandemic

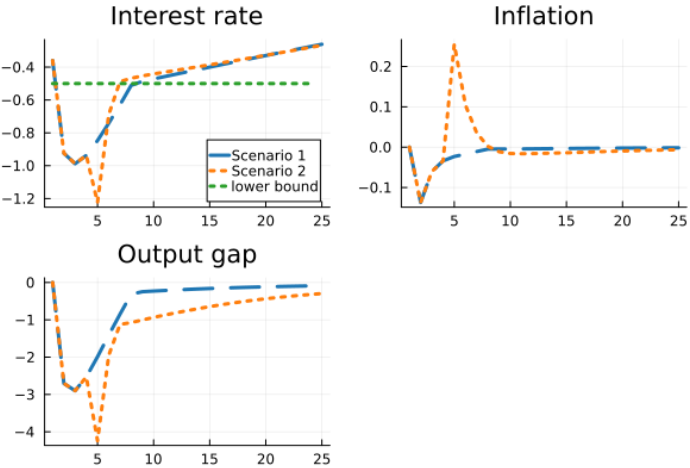


Demand shock in different regions of the Phillips curve

A demand shock increases inflation relatively more and output relatively less when inflation is higher



Supply shock when interest rate is at the lower bound



Scenario 1: the economy is hit by a demand shock that brings the interest rate at the ELB
Scenario 2: in addition, at period 5, a supply shock pushes inflation up

Other applications

- Guarnieri (2024) tests fiscal dominance hypothesis in the US:
  - finds that concerns about debt sustainability affect monetary policy
- Work in progress:
  - Testing the effectiveness of the Maastricht treaty (with Bonomolo, de Ferra and Romei)
  - Disentangling the effects of alternative unconventional monetary policies (with Ikeda and Shintani)

Conclusion

- Endogenous regime switching can be modelled via nonlinear structural models of the form

$$f_0(z_t) = c + \sum_{i=1}^{k} f_i(z_{t-i}) + u_t$$

where $f_i(\cdot)$ are of piecewise affine form

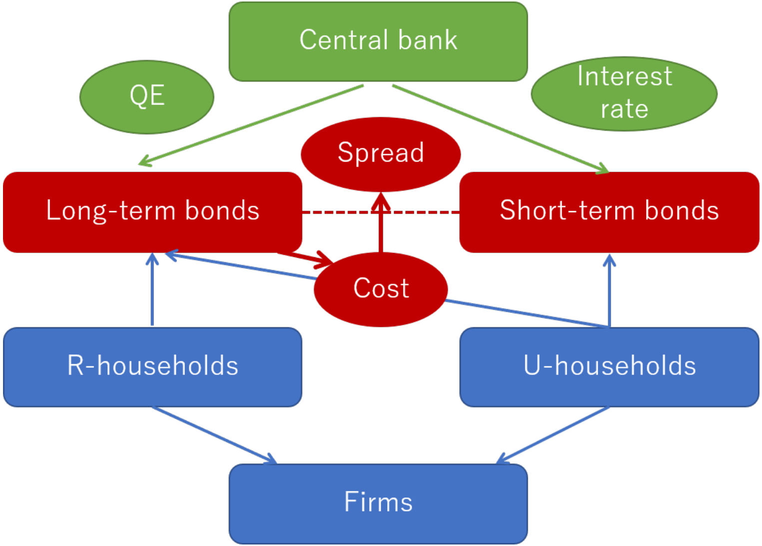
- It can capture nonlinearities and improve identification of causal effects
- Efficient estimation via Julia package EndoRSE

Theoretical model of unconventional policy

Ikeda et al (2024, AEJM)

- 3-equation New Keynesian model
  - IS, PC, Taylor rule
- Bond market segmentation makes QE effective
- Two types of FG:
  - Reifschneider & Williams (2000, JMCB)
  - Debortoli et al. (2019, Macro Annual)
- Both types of UMP can be captured by shadow rate

A New Keynesian model of UMP



SVAR representation

1. Under irrelevance hypothesis IRF can be obtained from linear (single-regime) SVAR in
   - inflation, output gap and the shadow rate
   - UMP is fully effective in overcoming ZLB constraint

2. If IH doesn't hold
   - There is a single parameter ξ that characterizes effectiveness of UMP
     - Combines both FG and QE
     - When $\xi = 1$, UMP is fully effective (irrelevance hypothesis holds)
   - When $\xi < 1$, and agents are boundedly rational, solution is piecewise linear SVAR in
     - inflation, output gap, and shadow rates
     - a special case of CKSVAR